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Non-relativistic M-Theory solutions based on Kähler-Einstein spaces

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Abstract

We present new families of non-supersymmetric solutions of $D = 11$ supergravity with non-relativistic symmetry, based on six-dimensional Kähler-Einstein manifolds. In constructing these solutions, we make use of a consistent reduction to a five-dimensional gravity theory coupled to a massive scalar and vector field. This theory admits a non-relativistic CFT dual with dynamical exponent $z = 4$, which may be uplifted to $D = 11$ supergravity. Finally, we generalise this solution and find new solutions with various z, including $z = 2$.

1 Introduction

Over the past year, non-relativistic conformal (NRC) field theories have attracted a lot of attention, primarily driven by the prospect of tailoring the AdS/CFT correspondence so that it may be used as a tool to describe condensed matter systems in a laboratory environment. These systems are described by Schrödinger symmetry, which is a non-relativistic version of conformal symmetry. The corresponding algebra is generated by Galilean transformations, an anisotropic scaling of space, $\mathbf{x} \to \lambda \mathbf{x}$, and time, $x^+ \to \lambda^z x^+$, where $z > 0$ is a real number usually referred to as the *dynamical exponent*, and an additional special conformal transformation when $z = 2$. For NRC field theories with one time and d spatial dimensions, the corresponding symmetry algebra will be denoted $\mathrm{Sch}_z(1, d)$.

Gravity duals for NRC field theories were initially proposed in [1, 2] and were subsequently embedded in type IIB in [3, 4, 5] and $D = 11$ supergravity in [6]. The IIB solutions of [3, 4, 5] with $z = 2$ are obtained by coordinate transformations which deform the three-form flux, but in the process break supersymmetry. Other techniques that have been employed in the construction of NRC gravity duals in type IIB and $D = 11$ supergravity include metric deformations [7] and uplift of suitable solutions of the lower dimensional theories to which the $D = 10, 11$ supergravities on Sasaki-Einstein manifolds consistently truncate [5, 6]. Some solutions obtained by these two methods do preserve supersymmetry [7, 8]. Solutions pursued via uplift turn out to permit only set dynamical exponents, whereas more general constructions, still based on Sasaki-Einstein spaces [8, 9, 10], allow for richer classes of solutions with many different values of z, including $z = 2$. For a selection of other works on gravity duals of NRC field theories in various dimensions, both supersymmetric and non-supersymmetric, see [11].

In all these cases, the $D = 10$ or $D = 11$ metric dual to an NRC field theory in spatial dimension d corresponds to a deformation of a given D–dimensional solution containing $(d + 3)$-dimensional Anti-de Sitter space, that breaks the original AdS_{d+3} isometry $so(2, d + 2)$ down to its $\mathrm{Sch}_z(1, d)$ subalgebra. The purpose of this paper is to obtain $D = 11$ supergravity solutions with $\mathrm{Sch}_z(1, 2)$ symmetry, associated to the $AdS_5 \times KE_6$ class of $D = 11$ supergravity solutions with KE_6 a six-dimensional Kähler-Einstein space of positive curvature [12, 13]. Interestingly enough, despite the lack of supersymmetry of the general $AdS_5 \times KE_6$ solution[1] for arbitrary KE_6, the special case when KE_6 is CP^3 has recently been shown to be classically stable

[1] See [14] for the classification of the superymmetric M-Theory solutions containing AdS_5.

[15]. We expect our $\mathrm{Sch}_z(1,2)$–invariant solutions, dual to NRC field theories in spatial dimension $d = 2$, to inherit the non-supersymmetric character of the original $AdS_5 \times KE_6$ solutions.

As mentioned earlier, the first examples of gravitational solutions dual to NRC field theories were found in lower-dimensional theories of gravity coupled to a massive vector field [1]. One benefit of much recent work on consistent Kaluza-Klein (KK) truncations [16, 17, 18] is that these solutions may be uplifted to type IIB [5] and $D = 11$ supergravity settings [6]. In a similar fashion, we will first show, in section 2, that there exists a consistent KK truncation of $D = 11$ supergravity on KE_6 to a $D = 5$ theory involving a massive vector and a massive scalar. We subsequently uplift, in section 3, a solution to the $D = 5$ theory to eleven-dimensions to find a new M-Theory solution with dynamical exponent $z = 4$. In section 4 we perform a generalisation to a class of NRC solutions obtained as deformations of the original $AdS_5 \times KE_6$ solution that, in general, cannot be obtained from uplift. In this class, we will find new $\mathrm{Sch}_z(1,2)$–invariant M-Theory solutions with different dynamical exponents z, including $z = 2$. Like the analog constructions in [7, 8, 9, 10], the metric of all these solutions will maintain the KE_6 part of the original $AdS_5 \times KE_6$. Further generalisations should be possible allowing for more general internal geometries [19].

The $AdS_5 \times KE_6$ geometries that we take as starting point for our analysis are solutions to the equations of motion of $D = 11$ supergravity,

$$dG_4 = 0 \ , \tag{1.1}$$

$$d *_{11} G_4 + \tfrac{1}{2} G_4 \wedge G_4 = 0 \ , \tag{1.2}$$

$$R_{AB} = \tfrac{1}{12} G_{AC_1C_2C_3} G_B{}^{C_1C_2C_3} - \tfrac{1}{144} g_{AB} G_{C_1C_2C_3C_4} G^{C_1C_2C_3C_4} = 0 \ , \tag{1.3}$$

with metric and four-form given, respectively, by

$$ds^2_{11} = ds^2(AdS_5) + ds^2(KE_6), \tag{1.4}$$

$$G_4 = cJ \wedge J \ . \tag{1.5}$$

Here, c is a constant, J is the Kähler form on KE_6, and the metrics $g_{\mu\nu}$ and g_{mn} for AdS_5 and KE_6, respectively, are normalised so that their with Ricci tensors are

$$R_{\mu\nu} = -2c^2 g_{\mu\nu}, \quad R_{mn} = 2c^2 g_{mn}. \tag{1.6}$$

Note. While we were in the process of completing this paper, [20] appeared which, although supersymmetric in the main, section 5 therein has some overlap with our analysis.

2 Consistent truncation of $D = 11$ supergravity on KE_6

For every general supersymmetric solution $AdS_n \times_w M_{D-n}$, where $\times_w$ denotes warped product, of a D-dimensional supergravity theory, there exists a consistent truncation of the D-dimensional theory down to a suitable n-dimensional pure, massless gauged supergravity [16, 17, 18]. For supersymmetric Freund-Rubin backgrounds, the massive supermultiplet containing the breathing mode of the internal space M_{D-n} can also be retained consistently, together with the supergravity multiplet [6]. In all these cases, the G-structure on M_{D-n} specified by supersymmetry plays a crucial role in constructing the KK ansatz which describes the embedding of the retained n-dimensional fields into the D-dimensional ones. In the case at hand here, despite the lack of supersymmetry of the $AdS_5 \times KE_6$ background (1.4), (1.5), the Kähler form J of KE_6 will still allow us to build a KK ansatz that consistently includes massive modes, along the lines of [6].

At any rate, there is an argument about which modes one should expect to be able to keep in the truncation of $D = 11$ supergravity on KE_6. Consider first the particular case when the internal KE_6 is CP^3, which has isometry group $SU(4)$, and for which the KK spectrum is explicitly known [15]. Following [21], one should be able to truncate consistently the KK tower of $D = 11$ supergravity on CP^3 to its $SU(4)$ singlet sector. This contains the massless graviton, one massive real scalar and one massive real vector [15], both with mass $12c^2$. Now, it is precisely the singlet character of these modes under the relevant $SU(4)$ symmetry of the particular $KE_6 = CP^3$ that makes them expected to be universal for all KE_6 spaces. We can thus predict a consistent truncation of $D = 11$ supergravity on *any* KE_6 to a $D = 5$ theory with the field content quoted above. In particular, no massless vector that could enter the $D = 5$ $N = 2$ supergravity multiplet along with the metric should be expected to survive the truncation, so the resulting $D = 5$ theory should not correspond to a supergravity[2].

Without much further ado, consider the following KK ansatz

$$ds^2_{11} = ds^2_5 + e^{2U} ds^2(KE_6), \tag{2.1}$$

$$G_4 = H_4 + H_2 \wedge J + cJ \wedge J\,, \tag{2.2}$$

[2]This is to be constrasted with the analog situation for skew-whiffed Freund-Rubin backgrounds: in spite of also breaking all supersymmetry, they do allow for a consistent truncation to a supergravity theory [6].

where U, H_4 and H_2 are, respectively, a scalar (the breathing mode of the internal KE_6), a four-form and a two-form on the external five-dimensional spacetime, with line element ds_5^2, and J is again the Kähler form on KE_6. By choosing the coefficient in the $J \wedge J$ term to be the same constant c that appears in the background flux (1.5) we are anticipating that this coefficient cannot be turned into a dynamical $D = 5$ field without violating the $D = 11$ Bianchi identity for G_4. Also, one could have tried to add to the KK ansatz (2.2) terms involving the holomorphic (3,0)-form Ω defining the complex structure on KE_6, but it is unclear how to deal with those terms when plugging the ansatz into the $D = 11$ equations of motion.

The KK ansatz (2.1), (2.2) reduces to the background solution (1.4), (1.5) for $U = H_4 = H_2 = 0$, $ds_5^2 = ds^2(AdS_5)$. More generally, direct substitution of (2.1), (2.2) into (1.1)–(1.3) shows that the KK ansatz also solves the $D = 11$ supergravity field equations provided the $D = 5$ fields satisfy

$$dH_4 = 0\,, \tag{2.3}$$

$$dH_2 = 0\,, \tag{2.4}$$

$$d(e^{6U} * H_4) + 6cH_2 = 0\,, \tag{2.5}$$

$$d(e^{2U} * H_2) + 2cH_4 + H_2 \wedge H_2 = 0\,, \tag{2.6}$$

$$d(e^{6U} * dU) + 2c^2(e^{-2U} - e^{4U})\mathrm{vol}_5 - \tfrac{1}{6}e^{6U} H_4 \wedge *H_4 = 0\,, \tag{2.7}$$

$$\begin{aligned} R_{\alpha\beta} \;=\; & -2c^2 e^{-8U}\eta_{\alpha\beta} + 6\left(\nabla_\beta\nabla_\alpha U + \partial_\alpha U \partial_\beta U\right) + \tfrac{3}{2}e^{-4U}\left(H_{\alpha\lambda}H_\beta{}^\lambda - \tfrac{1}{6}\eta_{\alpha\beta}H_{\lambda\mu}H^{\lambda\mu}\right) \\ & + \tfrac{1}{12}\left(H_{\alpha\lambda\mu\nu}H_\beta{}^{\lambda\mu\nu} - \tfrac{1}{12}\eta_{\alpha\beta}H_{\lambda\mu\nu\rho}H^{\lambda\mu\nu\rho}\right)\,. \end{aligned} \tag{2.8}$$

All the dependence on the internal KE_6 drops out, leaving fully-fledged $D = 5$ equations of motion for the $D = 5$ fields. This shows the consistency of the truncation.

We can now introduce the Lagrangian of the $D = 5$ theory and work out the masses of the various fields. First of all, the Bianchi identities (2.3), (2.4) for H_4 and H_2 can be trivially solved by introducing a three-form and a one-form potential such that

$$H_4 = dB_3\,, \tag{2.9}$$

$$H_2 = dB_1. \tag{2.10}$$

The Lagrangian that gives rise to the $D = 5$ equations of motion (2.5)–(2.8) upon

variation of B_3, B_1, U and the $D = 5$ metric $g_{\mu\nu}$ can then be worked out. It reads

$$\begin{aligned} \mathcal{L} \;&=\; e^{6U} R \,\mathrm{vol}_5 + 30 e^{6U} dU \wedge *dU - \tfrac{1}{2} e^{6U} H_4 \wedge *H_4 - \tfrac{3}{2} e^{2U} H_2 \wedge *H_2 \\ &\quad + 6c^2 \left(2e^{4U} - e^{-2U}\right) \mathrm{vol}_5 - B_1 \wedge \left(6cH_4 + H_2 \wedge H_2\right) \;, \end{aligned} \tag{2.11}$$

or, in terms of the Einstein frame metric $\bar{g}_{\mu\nu} = e^{4U} g_{\mu\nu}$,

$$\begin{aligned} \mathcal{L}_{\text{Einstein}} \;&=\; \bar{R}\, \bar{\mathrm{vol}}_5 - 18 dU \wedge \bar{*} dU - \tfrac{1}{2} e^{12U} H_4 \wedge \bar{*} H_4 - \tfrac{3}{2} H_2 \wedge \bar{*} H_2 \\ &\quad + 6c^2 \left(2e^{-6U} - e^{-12U}\right) \bar{\mathrm{vol}}_5 - B_1 \wedge \left(6cH_4 + H_2 \wedge H_2\right) \;, \end{aligned} \tag{2.12}$$

with barred quantities referring to the Einstein frame metric.

It is useful to dualise B_3 into a scalar B. In order to do this, define $H_5 = dH_4$ and add the piece

$$\mathcal{L}' = -BH_5 \tag{2.13}$$

to the Lagrangian (2.12). Integrating out H_4 we find that it is now given as

$$H_4 = -e^{-12U} \bar{*} H_1 \;, \tag{2.14}$$

where we have found it convenient to define

$$H_1 = dB - 6cB_1 \;. \tag{2.15}$$

Substituting this back into $\mathcal{L}_{\text{Einstein}} + \mathcal{L}'$ we find the dual Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{dual}} \;&=\; \bar{R}\, \bar{\mathrm{vol}}_5 - 18 dU \wedge \bar{*} dU - \tfrac{1}{2} e^{-12U} H_1 \wedge \bar{*} H_1 - \tfrac{3}{2} H_2 \wedge \bar{*} H_2 \\ &\quad + 6c^2 \left(2e^{-6U} - e^{-12U}\right) \bar{\mathrm{vol}}_5 - B_1 \wedge H_2 \wedge H_2 \;. \end{aligned} \tag{2.16}$$

The masses of the $D = 5$ fields can now be computed by expanding the Lagrangian (2.16) about the AdS_5 vacuum, keeping up to quadratic terms. Doing this, for U and B_1 we find

$$m_U^2 = m_{B_1}^2 = 12c^2 \;, \tag{2.17}$$

while B (the scalar dual to B_3) is just a Stückelberg field that can be gauged away to give B_1 its mass. As anticipated, the $D = 5$ theory obtained upon consistent KK truncation of $D = 11$ supergravity on KE_6, and described by the Lagrangian (2.12) or (2.16), contains the $D = 5$ metric, one massive scalar and one massive vector with mass (2.17). When $KE_6 = CP^3$, the $SU(4)$–neutrality (table 2 of [15]) and the masses (tables 3 and 4 of [15]) of U and B_1 show that these are the modes in the $k = 0$ level of the $(k + 3)(k + 4)c^2$ towers of real scalars and real one-forms, respectively.

We are interested in solutions to the $D = 5$ field equations (2.3)–(2.8) displaying NRC symmetry. Rather than working with the full theory, we will consider a suitable further truncation. There are three further consistent truncations, apparently no longer explained by a group theory argument as the one above. The first is obtained by setting $H_4 = H_2 = 0$, leaving only the five-dimensional metric and the breathing mode U. The second, leading to five-dimensional General Relativity with a cosmological constant, is trivially obtained by insisting on $H_4 = H_2 = 0$ and further setting $U = 0$. The third, which is the one we are interested in, will be described in the next section.

3 NRC solutions from uplift

It is consistent with the $D = 5$ equations of motion to set $H_4 = 6ce^{-6U} * B_1$, where the Hodge dual here refers again to the metric appearing in the Lagrangian (2.11), and B_1 is defined in (2.10) . Rather than a further truncation, this just corresponds to gauging away B_3 or, alternatively, the Stückelberg scalar B, as can be seen from equations (2.14), (2.15) . The third possible further truncation referred to above is obtained (having gauged away B_3) by further setting $U = 0$ (and, thus, $H_4 = 6c * B_1$) while restricting B_1 to light-like configurations,

$$B_1 \wedge *B_1 = 0 \ , \quad H_2 \wedge H_2 = 0 \ . \tag{3.1}$$

In this case, the equations of motion (2.5)–(2.8) reduce to (3.1) together with

$$d * H_2 + 12c^2 * B_1 = 0 \ , \tag{3.2}$$

$$R_{\alpha\beta} = -2c^2 \eta_{\alpha\beta} + \tfrac{3}{2} H_{\alpha\lambda} H_{\beta}{}^{\lambda} + 18c^2 B_\alpha B_\beta \tag{3.3}$$

(with $H_2 = dB_1$). Indeed, setting $U = 0$ and $H_4 = 6c * B_1$, equation (2.5) is identically satisfied; equations (2.6) and (2.7) reduce, respectively, to the second and first conditions in (3.1); equation (2.3) is obtained by differentiating (3.2); and, finally, the Einstein equation (2.8) reduces to (3.3).

The equations of motion (3.2), (3.3) can be derived from the Lagrangian[3]

$$\mathcal{L} = R\ \mathrm{vol}_5 + 6c^2 \mathrm{vol}_5 - \tfrac{3}{2} H_2 \wedge *H_2 - 18c^2 B_1 \wedge *B_1\ , \tag{3.4}$$

which was argued in [1] to allow for solutions with metric displaying Schrödinger symmetry. These solutions should be supported by a light-like massive vector of the form $B_1 \propto r^z dx^+$ (see [5]), where z is the dynamical exponent, thus immediately satisfying (3.1). Specifically, we look for solutions to (3.1), (3.2), (3.3) of the form

$$\begin{aligned} ds_5^2 &= -\alpha^2 r^{2z} (dx^+)^2 + \frac{2}{c^2 r^2} dr^2 + \frac{2}{c^2} r^2 \left(-dx^+ dx^- + dx_1^2 + dx_2^2 \right)\ , \\ B_1 &= \beta r^z dx^+. \end{aligned} \tag{3.5}$$

where α, β and the dynamical exponent z are constants to be determined. The configuration (3.5) does satisfy the conditions (3.1) and turns out to also solve the equations (3.2), (3.3) provided that

$$z(z+2) \;=\; 24\ , \tag{3.6}$$

$$\alpha^2 (z^2 - 1) \;=\; \beta^2 (\tfrac{3}{4} z^2 + 18). \tag{3.7}$$

Thus, as in [5], we indeed find solutions for $z = 4$ (and $\beta = \frac{\alpha}{\sqrt{2}}$) and $z = -6$ (and $\beta = \frac{\alpha\sqrt{7}}{3}$). By convention $z > 0$, so we ignore the latter possibility.

The $z = 4$ solution can now be uplifted to $D = 11$ with the help of the KK ansatz (2.1), (2.2). We find

$$\begin{aligned} ds_{11}^2 &= -\alpha^2 r^8 (dx^+)^2 + \frac{2}{c^2} \frac{dr^2}{r^2} + \frac{2}{c^2} r^2 \left(-dx^+ dx^- + dx_1^2 + dx_2^2 \right) + ds^2(KE_6)\ , \\ G_4 &= 12 \tfrac{\alpha}{c^2} r^5 dx^+ \wedge dr \wedge dx_1 \wedge dx_2 - 2\sqrt{2} \alpha r^3 dx^+ \wedge dr \wedge J + cJ \wedge J\ . \end{aligned} \tag{3.8}$$

This is a new (non-supersymmetric) M-Theory solution dual to a NRC field theory in spatial dimension $d = 2$ with dynamical exponent $z = 4$. One can generalise this solution and consider more general ansatze for $D = 11$ supergravity solutions dual to $d = 2$ non-relativistic conformal field theories with dynamical exponent z, where the internal directions still correspond to a KE_6 space. We now turn to this point.

[3]This $D = 5$ theory, with even the same mass for the vector B_1 if we choose $c = \sqrt{2}$, was first discussed in section 4.2 of [5], but the $D = 5$ parent theories with Lagrangian (2.16) above and (4.21) of [5] are very different. As in [5, 6], the Lagrangian (3.4) only reproduces the equations (3.2), (3.3) and not the light-like condition (3.1). Since (3.1), (3.2), (3.3) can be consistently obtained upon truncation of $D = 11$ supergravity on KE_6, any choice of five-dimensional metric and lightlike B_1 (thus subject to (3.1)) which also solves the equations of motion (3.2), (3.3) that derive from the Lagrangian (3.4), can be safely uplifted to $D = 11$.

4 Some generalisations

As we have just mentioned, the $D = 11$ solution (3.8) is locally invariant under $\mathrm{Sch}_4(1,2)$. In particular, the scale invariance acts on coordinates as [2]

$$(x^+, x^-, x_i, r) \to (\lambda^z x^+, \lambda^{2-z} x^-, \lambda x_i, \lambda^{-1} r) , \qquad i = 1, 2 \tag{4.1}$$

(with $z = 4$ in (3.8)), while leaving the KE_6 coordinates unchanged. Following [7, 8], we can generalise the metric in (3.8) as:

$$\begin{aligned} ds^2_{11} \;=\; & \frac{2}{c^2}\Big[- f_0 r^{2z} (dx^+)^2 - r^2 dx^+ (dx^- + r^{z-2} C_1) + \frac{1}{r^2} dr^2 \\ & + r^2 \big(dx_1^2 + dx_2^2 \big) \Big] + ds^2(KE_6) , \end{aligned} \tag{4.2}$$

where C_1 is a one-form and f_0 a function, both of them defined on the internal KE_6. Both C_1 and $r^{2z} f_0$, serve the same role of breaking the $SO(2,4)$ isometry of the original $AdS_5 \times KE_6$ metric (1.4) down to $\mathrm{Sch}_z(1,2)$.

An ansatz for the accompanying four-form flux may be constructed by considering the forms invariant under $Sch_z(1,2)$ symmetry (see [22]), though the equations of motion constrain the candidate forms. The specific ansatz we then consider for the four-form flux is

$$G_4 \;=\; -\tfrac{1}{z+2} d(\mu_0 r^{z+2} dx^+ \wedge dx^1 \wedge dx^2) - \tfrac{1}{z} d(\mu_2 \wedge r^z dx^+) + cJ \wedge J , \tag{4.3}$$

where, in general, μ_0 is a function and μ_2 a two-form, both defined on KE_6. The latter can be taken to be proportional to the Kähler form on KE_6, as for the uplifted $z = 4$ solution (3.8), but other choices are also possible (see subsection 4.2 below). Indeed, the solution (3.8) is recovered from (4.2), (4.3) by setting $C_1 = 0$, $f_0 = \frac{1}{2} c^2 \alpha^2$, $\mu_0 = \frac{12\alpha}{c^2}$ and $\mu_2 = -2\sqrt{2}\alpha J$, for some constant α. More generally, the non-trivial mixing of external and KE_6 coordinates in the metric (4.2) will prevent it from being obtainable as the uplift of any $D = 5$ metric. The requirement that (4.2), (4.3) do solve the equations of motion (1.1)–(1.3) of $D = 11$ supergravity leads to restrictions and relations for f_0, C_1, μ_0 and μ_2. In the following, we will spell out several interesting cases.

4.1 A solution with $z = 2$

We can find a $D = 11$ supergravity solution with dynamical exponent $z = 2$ by setting, for some constant α, $f_0 = \frac{13\alpha}{4c^4}$, choosing C_1 such that $dC_1 = \alpha J$, while writing $\mu_0 = \frac{12\alpha\sqrt{2}}{c^5}$, $\mu_2 = -\frac{2\alpha}{c^3}$ so that the flux (4.3) reads

$$G_4 = \tfrac{12\alpha\sqrt{2}}{c^5} r^3 dx^+ \wedge dr \wedge dx_1 \wedge dx_2 - \tfrac{2\alpha}{c^3} r dx^+ \wedge dr \wedge J + cJ \wedge J . \tag{4.4}$$

A generalisation of this solution appeared previously in [20], where the internal space is a variant of CP^3 [13].

4.2 A class of solutions with $z \geq \sqrt{3}$

Setting $C_1 = 0$ in the metric (4.2) and $\mu_0 = 0$, $\mu_2 = 0$ in (4.3) (which takes the flux back to its background value (1.5)), some calculation reveals that the resulting combination of metric and four-form provides a solution of $D = 11$ supergravity if f_0 is an eigenfunction of the Laplacian $\Delta_{KE} \equiv *d*d + d*d*$ on KE_6 with eigenvalue $2(z^2-1)c^2$:

$$\Delta_{KE} f_0 = 2(z^2-1)c^2 f_0. \tag{4.5}$$

This class of solutions thus provides a $D = 11$ counterpart of the Type IIB solutions first discussed in [7].

For the particular case $KE_6 = CP^3$, these eigenvalues are $k(k+3)c^2$, $k = 0, 1, \ldots$, with the corresponding eigenfunctions transforming in the $(k0k)$ irrep of $SU(4)$ [23, 15]. Ruling out $k = 0$, which just corresponds to a space locally isometric to $AdS_5 \times KE_6$, we have a sequence of families of solutions with dynamical exponents

$$z_k = \sqrt{1 + \tfrac{1}{2}k(k+3)}\ , \quad k = 1, 2 \ldots\ , \tag{4.6}$$

thus obeying the bound

$$z_k \geq \sqrt{3}\ . \tag{4.7}$$

For each $k = 1, 2, 3 \ldots$, this class contains a family of $\dim(k0k) = 15,\ 84,\ 300, \ldots$ supergravity solutions with the dynamical exponent z_k in (4.6).

As noted in [7], this class of solutions should be unstable. Stability could be restored in [7] by appropriately turning on fluxes. We can try to do the same here by setting, for simplicity, μ_2 to be proportional to the Kähler form J. In this case, only for $z = 4$ do we find a solution with metric (4.2) (with $C_1 = 0$), supported by the flux

$$G_4 = \alpha r^5 dx^+ \wedge dr \wedge dx_1 \wedge dx_2 - \tfrac{\alpha c^2}{3\sqrt{2}} r^3 dx^+ \wedge dr \wedge J + cJ \wedge J\ , \tag{4.8}$$

for any constant α. In this case, f_0 gets shifted by a positive term proportional to α^2, which can be tuned to render the solution stable [7]. The shifted f_0 still fulfils (4.5), now with eigenvalue $30c^2$, corresponding to $z = 4$. We are unaware, however, of any KE_6 space for which this eigenvalue is permissible.

Alternatively, following [8, 9, 10], rather than setting μ_2 to be proportional to the Kähler form, one may take it to be primitive and transverse[4]. Setting, for convenience, $\mu_0 = C_1 = 0$, a calculation shows that the configuration (4.2), (4.3) is a solution to $D = 11$ supergravity provided

$$\begin{aligned} \Delta_{KE} f_0 + 2(z^2-1)c^2 f_0 &= \frac{c^4}{4}|\mu_2|^2 + \frac{c^2}{2z^2}|d\mu_2|^2, \\ \Delta_{KE}\mu_2 &= \tfrac{1}{2}z(z+2)c^2\mu_2, \end{aligned} \tag{4.9}$$

where $|\mu_2|^2 = \frac{1}{2!}\mu_{2\ ab}\mu_2^{ab}$, etc. Now, f_0 has devolved the Laplacian eigenvector character upon μ_2, which corresponds to a two-form eigenfunction with eigenvalue $\frac{1}{2}z(z+2)c^2$. In the special case $KE_6 = CP^3$, the eigenvalues of the Laplacian acting on transverse, primitive $(1,1)$–forms (respectively, $(2,0)$–forms) are $(k+2)(k+3)c^2$ (respectively, $(k+3)(k+4)c^2$), for $k = 0, 1, \ldots$ [23, 15]. We thus see that solutions to (4.9) correspond to NRC gravity duals with dynamical exponents bounded below by $z \geq -1 + \sqrt{13}$ (respectively, $z \geq 4$), if μ_2 is a chosen to be (the real part of) a $(1,1)$–form (respectively, $(2,0)$–form). See [10] for a discussion of a solving technique for systems of equations like (4.9). It would be interesting to study the stability of this class of solutions.

5 Final comments

We have constructed solutions of $D = 11$ supergravity dual to NRC field theories in 2 spatial dimensions and with different values of the dynamical exponent z. They correspond to suitable deformations of the class of solutions $AdS_5 \times KE_6$, that break the $SO(2,4)$ symmetry down to its Schrödinger subalgebra $\mathrm{Sch}_z(1,2)$. Important insight was obtained by first dealing with a simpler, particular solution with $z = 4$. Specifically, $D = 11$ supergravity reduced on the internal KE_6 truncates consistently to a $D = 5$ gravity theory involving a massive vector. A suitable solution of this theory, with $z = 4$, was found and subsequently uplifted to eleven-dimensions. We also discussed a more general class of $D = 11$ supergravity solutions, locally invariant under $\mathrm{Sch}_z(1,2)$, that contains this solution, along with other examples that can no longer be obtained upon uplift. We are able to find explicitly a solution with $z = 2$, a class of solutions with dynamical exponents $z \geq \sqrt{3}$, and implicitly, solutions with $z \geq -1 + \sqrt{13}$ and $z \geq 4$.

[4]A (p,q)–form $Y^{p,q}$ on a Kähler space is said to be primitive if its contraction with the Kähler form vanishes, $J^{mn}Y^{p,q}_{mn\ldots} = 0$, and transverse if $*d*Y^{p,q} = 0$.

The Schrödinger algebra $\mathrm{Sch}_z(1,d)$ is not the only NRC symmetry one may consider. In fact, there also exists a conformal version of the Galilean algebra that, unlike $\mathrm{Sch}_z(1,d)$, can be obtained as an Inönü-Wigner contraction of the relativistic conformal algebra $so(2,d+2)$. Some issues regarding the Galilean conformal algebra have been recently discussed, including its supersymmetrisation [24, 25, 26] and its implementation, both in the dual field theories and the gravity bulk [27, 28]. As pointed out in [28], a drawback of backgrounds with this conformal Galilean symmetry is that, in contrast to $\mathrm{Sch}_z(1,d)$–invariant ones, their metrics exhibit a non-Lorentzian signature. While this would require better understanding, progress on the way NRC symmetries are implemented in gravity duals may be achieved by a systematic characterisation [19] of Type IIB and M-Theory backgrounds with $\mathrm{Sch}_z(1,d)$ symmetry, for generic values of z and d.

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